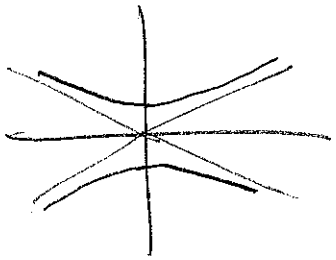
Find the **rectangular** equation of the hyperbola with foci $(0, \pm 2)$ and asymptotes $y = \pm\frac{3}{4}x$. **SCORE: ____ / 20 PTS**



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$\frac{a}{b} = \frac{3}{4} \longrightarrow a = \frac{3}{4}b$$

$$a^2 + b^2 = 2^2$$

$$\left(\frac{3}{4}b\right)^2 + b^2 = 4$$

$$\frac{25}{16}b^2 = 4$$

$$b^2 = \frac{64}{25}$$

$$b = \frac{8}{5}$$

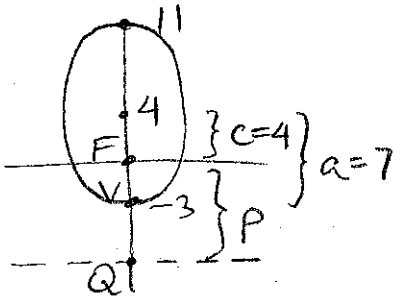
$$a = \frac{6}{5}$$

$$\frac{y^2}{\frac{36}{25}} - \frac{x^2}{\frac{64}{25}} = 1$$

OR

$$\frac{25y^2}{36} - \frac{25x^2}{64} = 1$$

Find the **polar** equation of the ellipse with vertices with **polar** co-ordinates $(11, \frac{\pi}{2})$ and $(3, \frac{3\pi}{2})$, and a focus at the pole. **SCORE: ____ / 20 PTS**



$$r = \frac{ep}{1 - e\sin\theta}$$

OR

$$11 = \frac{ep}{1 - e\sin\frac{\pi}{2}} \longrightarrow 11 = \frac{ep}{1-e}$$

$$3 = \frac{ep}{1 - e\sin\frac{3\pi}{2}} \longrightarrow 3 = \frac{ep}{1+e}$$

$$ep = 11 - 11e = 3 + 3e$$

$$8 = 14e$$

$$e = \frac{4}{7}$$

$$\frac{4}{7}p = 3 + 3\left(\frac{4}{7}\right) = \frac{33}{7}$$

$$p = \frac{33}{4}$$

CENTER $= \left(0, \frac{11-3}{2}\right)$

$= (0, 4)$

$$e = \frac{c}{a} = \frac{4}{7}$$

$$e = \frac{VF}{VQ} = \frac{3}{p-3} = \frac{4}{7}$$

$$21 = 4p - 12$$

$$p = \frac{33}{4}$$

$$r = \frac{\frac{4}{7}\cdot\frac{33}{4}}{1 - \frac{4}{7}\sin\theta} = \frac{33}{7 - 4\sin\theta}$$

Find the zeros of the polar equation $r = 1 + 2\sin 2\theta$ for $\theta \in [0, 2\pi)$. **SCORE: ____ / 15 PTS**

NOTE: You must solve this problem algebraically, NOT BY TRIAL & ERROR.

$0 = 1 + 2\sin 2\theta$ $\quad 0 \le \theta < 2\pi$

$\sin 2\theta = -\frac{1}{2}$ $\quad 0 \le 2\theta < 4\pi$

$2\theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$

$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

Convert the polar equation $r = 3 + \cos 2\theta$ to rectangular. **SCORE: ____ / 15 PTS**

$r = 3 + \cos^2\theta - \sin^2\theta$

$r = 3 + \frac{x^2}{r^2} - \frac{y^2}{r^2}$

$r^3 = 3r^2 + x^2 - y^2$

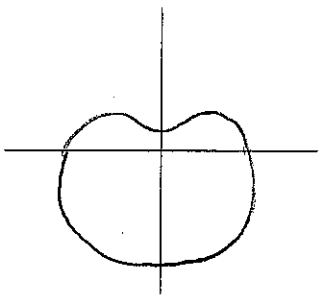
$(x^2+y^2)^{\frac{3}{2}} = 3x^2 + 3y^2 + x^2 - y^2$

$(x^2+y^2)^{\frac{3}{2}} = 4x^2 + 2y^2$

$(x^2+y^2)^3 = (4x^2 + 2y^2)^2$

Sketch the general shape, position and direction of the polar curve $r = 7 - 4\sin\theta$ on the axes on the right. **SCORE: ____ / 12 PTS**

NOTE: You do NOT need to find specific points. However, if the curve passes through the pole, it must be shown on the graph.



$1 < \left|\frac{7}{-4}\right| < 2 \rightarrow$ LIMACON WITH DIMPLE

SYMMETRIC OVER y-AXIS

LARGER SIDE DOWN

Consider the polar equation $r = 3 - \cos 2\theta$.

SCORE: ____ / 27 PTS

The following symmetry tests all fail: $(-r, \theta)$, $(-r, -\theta)$ and $(-r, \pi - \theta)$

[a] Run the other standard tests for symmetry for the polar equation, and summarize all conclusions in the table below.

$(r, -\theta)$: $r = 3 - \cos 2(-\theta)$

$r = 3 - \cos(-2\theta)$

$r = 3 - \cos 2\theta$

SYMMETRIC OVER POLAR AXIS

Type of symmetry	Conclusion
Over the pole	SYMMETRIC
Over the polar axis	SYMMETRIC
Over $\theta = \frac{\pi}{2}$	SYMMETRIC

$(r, \pi - \theta)$: $r = 3 - \cos 2(\pi - \theta)$

$r = 3 - \cos(2\pi - 2\theta)$

$r = 3 - [\cos 2\pi \cos 2\theta + \sin 2\pi \sin 2\theta]$

$r = 3 - \cos 2\theta$

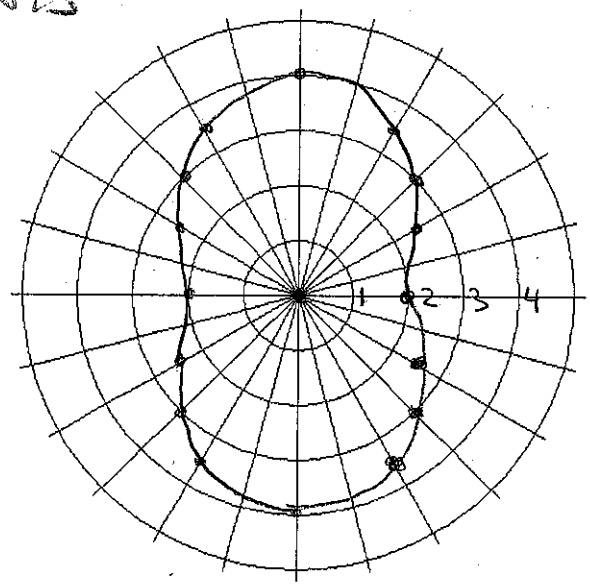
SYMMETRIC OVER $\theta = \frac{\pi}{2}$

AUTOMATICALLY SYMMETRIC OVER POLE

[b] Draw the graph of this polar equation by plotting points for as few θ – values as needed, and using symmetry to complete the graph. **List the polar co-ordinates (in ordered pair notation) of all points you needed to plot.**

ONLY NEED TO PLOT POINTS IN 1 QUADRANT

θ	r	
0	2	$(2, 0)$
$\frac{\pi}{6}$	2.5	$(2.5, \frac{\pi}{6})$
$\frac{\pi}{4}$	3	$(3, \frac{\pi}{4})$
$\frac{\pi}{3}$	3.5	$(3.5, \frac{\pi}{3})$
$\frac{\pi}{2}$	4	$(4, \frac{\pi}{2})$



Consider the conic with rectangular equation $9x^2+4y^2+72x-8y+4=0$. **SCORE: ____ / 20 PTS**

[a] Find the co-ordinates of the vertex/vertices.

$9x^2+72x+4y^2-8y=-4$

$9(x^2+8x)+4(y^2-2y)=-4$

$9(x^2+8x+16)+4(y^2-2y+1)=-4+144+4$

$9(x+4)^2+4(y-1)^2=144$

$\frac{(x+4)^2}{16}+\frac{(y-1)^2}{36}=1$ ELLIPSE

CENTER $(-4,1)$

VERTICES $(-4,1\pm6)$

$=(-4,7),(-4,-5)$

[b] Find the co-ordinates of the focus/foci.

$c^2=36-16=20$

$c=2\sqrt{5}$

FOCI $(-4,1\pm2\sqrt{5})$

Name the shapes of the following graphs. **SCORE: ____ / 21 PTS**

[a] the graph with polar equation $r=-5+2\cos\theta$

CONVEX LIMACON $\left|\frac{-5}{2}\right|>2$

[b] the graph with equation $8+4x+2x^2+18y-3y^2=0$

HYPERBOLA

[c] the graph with polar equation $\theta=2$

LINE

[d] the graph with polar equation $r=\dfrac{15}{7-5\cos\theta}$

ELLIPSE $e=\frac{5}{7}$

[e] the graph with polar equation $r=-7-7\cos\theta$

CARDIOID $\left|\frac{-7}{-7}\right|=1$

[f] the graph with equation $2+2x+3x^2+18y=0$

PARABOLA

[g] the locus of points in the plane that are one-sixth as far from $(4,1)$ as they are from $x=7$

P F D

ELLIPSE $PF=\frac{1}{6}PQ \rightarrow \frac{PF}{PQ}=\frac{1}{6}=e$